Introduction / Einführung:
Joasia Krysa
The significance of Note G is that it provides a description of method and a diagram of an algorithm for setting up the engine to compute the Bernoulli numbers. The diagram is widely referred to as the first computer program, and the notes the first expression of computer theory. Together, they can be considered what we would describe in contemporary terms as the software required to operate the hardware of Babbage’s machine, which did not yet exist. However, at the time there remained uncertainty over the significance of Babbage’s invention, and the future potential of computation per se. Note G, reproduced in full in this notebook from its first published version of 1843, contains Lovelace’s reservations in this respect.

It is desirable to guard against the possibility of exaggerated ideas that might arise as to the powers of the Analytical Engine. The Analytical Engine has no pretensions whatever to originate anything. It can do whatever we think fit to order it to perform. It can follow any analytical operation whatever. In fact the engine may be described as being the material expression of any indefinite function of any degree of generality and complexity. The operating mechanism can even be thrown into action independently of any object to operate upon.

While Note G advises caution as to the potential of the Engine, and computational machines more generally, to demonstrate independent thinking (artificial intelligence), Lovelace recognizes the particular significance of the Engine in marking a transition from calculation to general-purpose computing—from a machine merely able to tabulate numbers to a programmable universal machine capable of manipulating symbols according to rules and of generating anything at all, whether music, poetry, or images. In Note A she writes:

The distinctive feature of the machine is its use of punch cards for programming the Engine, adopted by Babbage after they were first introduced by Joseph Jacquard to instruct the loom to automate and regulate weaving patterns. The Analytical Engine, on the contrary, is not merely adapted for calculating any function whatever. In fact the engine has no pretensions whatever to being the material expression of any analytical relations or truths. Its province is to assist us in making available what we are already acquainted with.

In response to his question about the missing Note G, Lovelace writes: “There never was a Note G. I do not know why I chose H instead of G, & thus insulted the latter worthy letter.”1 In the final published version of the work, Note G never was a Note G. I do not know why I chose H instead of G, & thus insulted the Engine Inventor Charles Babbage.

There Never Was a Note G

In his letter to Ada Augusta Lovelace of July 2, 1843, Charles Babbage writes: “I like much the improved form of the Bernoulli Note but can judge of it better when I have the Diagram and Notation.”2 He is referring to the last in a set of notes written by Lovelace that interpreted the Analytical Engine, the first fully automatic and universal computer, invented by Babbage in 1834, although never actually completed during his lifetime. She appended these notes to her translation of an article written by Luigi Federico Menabrea after he had heard Babbage present a paper on the Engine. Her translation, together with her extensive notes (three times the length of the original article), were published in 1843 and signed A.A.L.2

In the same letter, Babbage recounts the order of the notes in preparation for submitting them to the publisher:

A Sent to Lady L. 
B With C.B. 
C Ditto 
D Sent to Lady L. 
E With C.B. 
F Retained by Lady L. 
G Where is it gone? 
H With C.B.

In response to the question about the missing Note G, Lovelace writes: “There never was a Note G. I do not know why I chose H instead of G, & thus insulted the latter worthy letter.”3 In the final published version of the work, Note H becomes Note G and subsequently, along with Babbage’s machine and the rest of Lovelace’s notes, a key reference point in the history of modern computing.4

1 | Letter from Charles Babbage to Ada Augusta Lovelace, July 2, 1843, typed copy, British Library, Additional AID 54089.
3 | Letter from Ada Augusta Lovelace to Charles Babbage, July 2, 1843, handwritten, British Library, Additional AID 37192.
4 | This came almost one hundred years in advance of the work of Konrad Zuse, Alan Turing, Howard Aiken, and Grace Hopper and the invention of the machines that came to symbolize the age of modern computing (Z1, Baby Manchester Computer, Harvard Mark I, and ENIAC).
5 | In Note G, Lovelace explains that her choice of Bernoulli numbers—a sequence of rational numbers—to demonstrate the computing powers of the Engine is “a rather complicated example.” (“Sketch of the Analytical Engine Invented by Charles Babbage”) [see note 2]. Note G, p. 724.”
6 | Ibid., p. 722.
7 | In time, this also became contestable. In 1950, Alan Turing, mathematician and computer scientist, wrote his seminal paper “Computing Machinery and Intelligence” to question the perceived limitations of machines for independent thinking (artificial intelligence), against the reservations expressed by Lovelace some hundred years earlier. He famously asked: “Can machines think?”
9 | Ibid., p. 696.
10 | Ibid., pp. 696, 694.
11 | Donald E. Knuth, The Art of Computer Programming (Reading, Mass.: Addison-Wesley Publishing Company, 1981 [orig. 1968]). It is also interesting to note that there is an object-oriented programming language named Ada in recognition of Lovelace’s contribution to programming.
12 | Geoffrey Batchen, “Electricity Made Visible,” in New Media Art: Context and Practice in the UK, 1994–2004, ed. Lucy Kimbell (London and Manchester: Arts Council England and Cornerhouse Publications, 2004), pp. 27–48. In 1999 Huiyin Alpaym made an embroidered canvas that connects many of those threads and named it “Lovelace.” He explained: “Lace, which denotes a completion of love, is remembered along with love, purveys meaning both in the sense of a bringer as well as in terms of embroidery. Besides this, lace is once again a borrowing from disparate references and is a send off to different contexts. Like the famous porno star Linda Lovelace, it is also a borrowing of the famous mathematician Ada Byron Lovelace whose name was given to an intelligence server: Talks, curtailing, erasing, removing, stitching… Fantasy deals with
As soon as I have brought flying to perfection, I have got a scheme about a . . . steamengine which, if ever I effect it, will be more wonderful than either steampackets or steamcarriages, it is to make a thing in the form of a horse with the steamengine in the inside so contrived as to move an immense pair of wings, fixed on the outside of the horse, in such a manner as to carry it up into the air while a person sits on its back.13

In her work, as well as in her life, Lovelace managed to combine scientific rationalism with subjective imagination, influenced by her experience of the Industrial Revolution and the many technological innovations at that time.14 However, in a departure from the discipline-based systems of thinking and acquiring knowledge reinforced by the industrial period, she strongly believed in the need for connecting all disciplines. This attempt to go beyond the separation of fields of knowledge has since become a common thread in contemporary thinking, as for instance in the work of cybernetician Heinz von Foerster, who argued that in an increasingly complex world, it is no longer possible to maintain traditional science as the dominant structure of thinking. Consequently, there is a shift toward what he described as “systemics,” an approach that sees things together in complex connections and interrelations.15 Lovelace’s term for this was “Poetical Science,” and her Note G anticipates the indefinite potential of machines to express complexity.16

As soon as I have brought flying to perfection, I have got a scheme about a . . . steamengine which, if ever I effect it, will be more wonderful than either steampackets or steamcarriages, it is to make a thing in the form of a horse with the steamengine in the inside so contrived as to move an immense pair of wings, fixed on the outside of the horse, in such a manner as to carry it up into the air while a person sits on its back.13

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Joasia Krysa (b. 1969) is a curator, academic, and Agent for dOCUMENTA (13).
Einführung

Joasia Krysa

Es gab nie eine Anmerkung G

In einem Brief an Ada Augusta Lovelace vom 2. Juli 1843 schreibt Charles Babbage: »Die verbesserte Form der Bernoulli-Anmerkung gefällt mir sehr gut, aber ich kann sie besser beurteilen, wenn ich das Diagramm und die Notation habe.« \(^1\) Was Babbage hier als Bernoulli-Anmerkung bezeichnet, ist die letzte in einer Reihe von Anmerkungen, in denen Lovelace die Analytical Engine (Analytische Maschine) interpretierte, den ersten vollautomatischen, universellen Computer, den Babbage 1834 erfunden hatte, der zu seinen Lebzeiten jedoch niemals ganz fertiggestellt wurde. Sie fügte diese Anmerkungen ihrer Übersetzung eines Artikels von Luigi Federico Menabrea hinzu, den dieser verfasste, nachdem er einen Vortrag von Babbage über die Maschine gehört hatte. Lovelaces Übersetzung erschien 1843 mit ihren ausführlichen Anmerkungen (die drei Mal so lang waren wie der Aufsatz) und war mit A.A.L. unterzeichnet.\(^2\)

Im gleichen Brief wiederholt Babbage die Reihenfolge der Anmerkungen, die zur Übermittlung an den Verleger in Vorbereitung waren:

A An Lady L. geschickt
B Bei C.B.
C Dito
D An Lady L. geschickt
E Bei C.B.
F Von Lady L. einbehalten
G Wo ist sie geblieben?
H Bei C.B.

In ihrer Antwort auf seine Frage nach der fehlenden Anmerkung G schreibt Lovelace: »Es gab nie eine Anmerkung G. Ich weiß nicht, warum ich H

---

\(^1\) Brief von Charles Babbage an Ada Augusta Lovelace, 2. Juli 1843, Schreibmaschinenabteilung, British Library, Additional ADD 54089.

Die Besonderheit der Maschine ist der Einsatz von Lochkarten, die zu programmierbaren universellen Maschinen, die in der Lage ist, Symbole nach bestimmten Regeln zu verarbeiten und auf diese Weise die unterschiedlichsten Dinge, wie etwa Musik, Dichtung oder Bilder, zu generieren. In Anmerkung A schreibt sie:  

"Die Analytical Engine hingegen ist nicht nur das, was es ist, sondern es ist ein durchgängiger Aspekt des zeitgenössischen Denkens, wie etwa im Werk des Kybernetikers Heinz von Foerster, der argumentierte, dass es in einer immer komplexer werdenden Welt nicht mehr möglich sei, die traditionellen Wissenschaften als vorherrschende Denkstruktur aufrechtzuerhalten. Daraus folgt eine Verschiebung zu einer Herangehensweise, die er als 'systemics' bezeichnet, und die in den komplexen Verbindungen und Wechselbeziehungen Dinge zusammen sieht. Lovelaces Begriff hierfür war 'Poetische Wissenschaft', und ihre Anmerkung G nimmt das unbegrenzte Potenzial von Maschinen, Komplexität auszudrücken, vorweg."
SKETCH
OF THE
ANALYTICAL ENGINE
INVENTED BY
CHARLES BABBAGE, Esq.

By L. F. MENABREA,
of Turin.
OFFICER OF THE MILITARY ENGINEERS.
WITH NOTES BY THE TRANSLATOR.

[Extracted from the 'Scientific Memoirs,' vol. iii.]

LONDON.
PRINTED BY RICHARD AND JOHN E. TAYLOR,
RED LION COURT, FLEET STREET.
1843.
highly important for some of the future wants of science in its manifold, complicated and rapidly-developing fields of inquiry, to arrive at.

Without, however, stepping into the region of conjecture, we will mention a particular problem which occurs to us at this moment in being an apt illustration of the one to which such an engine may be turned for determining that which human brains find it difficult or impossible to work out unerring. In the solution of the famous problem of the Three Bodies, there are, out of about 295 coefficients of lunar perturbations given by M. Clausen (Astro., Nachr. No. 408) as the result of the calculations by Burg, of two by Dumasco, and of one by Burchardt, fourteen coefficients that differ in the nature of their algebraic sign; and out of the remainder there are only 101 (of about one-half) that agree precisely both in signs and in amount. These discrepancies, which are generally small in individual magnitude, may arise either from an erroneous determination of the absolute coefficients in the development of the problem, or from discrepancies in the data deduced from observation, or from both causes combined. The former is the most ordinary source of error in astronomical computations, and this the engine would entirely obviate.

We might even invent laws for series or formula in an arbitrary manner, and set the engine to work upon them, and thus deduce numerical results which we might not otherwise have thought of obtaining. But this would hardly perhaps in any instance be productive of any great practical utility, or calculated to make higher than as a kind of philosophical amusement.

NOTE G.—Page 698.

It is desirable to guard against the possibility of exaggerated ideas that might arise as to the powers of the Analytical Engine. In considering any new subject, there is frequently a tendency, first, to overrate what we find to be already interesting or remarkable; and, secondly, by a sort of natural selection, to underestimate the true state of the case, when we do discover that our notions have surpassed those that were really tenable.

The Analytical Engine has no pretensions whatever to originate any thing. It can do whatever we know how to order it to perform. It can follow a series, but has no power of anticipating any analytical relations or truths. Its province is to assist us in making available what we have already worked out. This it is calculated to effect primarily and chiefly of course, through its executive faculties; but it is equally to exert an indirect and reciprocal influence on science in another manner. For, in so distributing and combining the truths and the formulæ of analysis, that they may become most easily and rapidly available to the mechanical combinations of the engine, the relations and the nature of many subjects in that science are necessarily thrown into new lights, and more profoundly investigated. This is a decidedly indirect, and a somewhat speculative, consequence of such an invention. It is however pretty evident, on general principles, that it is in devising for mathematical truths a new form in which to represent and throw themselves out for actual use, views are likely to be induced, which should again react on the more theoretical phases of the subject. There are in all extensions of human power, or additions to human knowledge, various collateral influences, besides the main and primary object aimed.

To return to the executive faculties of this engine: the question must arise in every mind, are they really even able to follow analysis in its wide extent? No reply, entirely satisfactory to all minds, can be given to this query, excepting the actual existence of the engine, and actual experience of its practical results. We will however sum up for each reader’s consideration the chief elements with which the engine works:

1. It performs the four operations of simple arithmetic upon any numbers whatever.

2. By means of certain arithmetical, and combinations (upon which we cannot enter within the restricted space which such a publication as the present may admit of), there is no limit either to the magnitudes of the numbers used, or to the number of quantities (either variables or constants) that may be employed.

3. It can combine these numbers and these quantities either algebraically or arithmetically, in relations unlimited as to variety, extent, or complexity.

4. It uses algebraic signs according to their proper laws, and develops the logical consequences of these laws.

5. It can arbitrarily substitute any formula for any other; effecting the first from the columns on which it is represented, and making the second appear in its stead.

6. It can provide for singular values. Its power of doing this is referred to M. Menabrea’s memoirs, page 685, where he mentions the passage of values through zero and infinity. The practicability of causing it arbitrarily to change its processes at any moment, as the occurrence of any specified contingency (of which its substitution of \( \sin \pi x + \frac{1}{2} \cos \pi x = \pm 1 \); for \( \cos \pi x + \cos \pi x = 1 \) explained in sect. 5. is in some degree an illustration), at once secures this point.

The subject of integration and of differentiation demands some notice. The engine can effect these processes in either of two ways:

First. We may order it, by means of the Operation and of the Variable-constants, to go through the various steps by which the required limit can be worked out for whatever function is under consideration.

Secondly. It may (if we know the form of the limit for this function in question) effect the integration or differentiation by direct * substitution.

* The engine cannot of course compute limits for perfectly simple and uncompounded functions, except in this manner. It is obvious that it has no power of representing or of manipulating with any but finite increments or decrements, and consequently that wherever the computation of limits or of any other function depends upon the direct introduction of quantities which either increase or decrease indefinitely, we are absolutely beyond the sphere of its power. Its nature and arrangement are remarkably adapted for taking into account all finite increments or decrements (however small or large), and for developing the true and logical consequences of terms or value dependent upon differences of this nature. The engine may indeed be considered as including the whole. It is the Colours of Finite Difference, many of whose theorems would be especially and beautifully fitted for development by its processes, and would offer peculiarly interesting considerations. We may mention, as an example, the calculation of the Numbers of Bernoulli by means of the Differences of Nilth.
We remarked in Note E, that any set of columns on which numbers are inscribed, represents merely a general function of the several quantities, and the special function have been impressed by means of the Operation and Variable cards. Consequently, if instead of requiring the value of the function, we require that of its integral, or of its differential coefficient, we have merely to order whatever particular combination of the ingredient quantities may constitute that integral or that coefficient. In $\alpha \infty$, for instance, instead of the quantities $V_0 V_1 V_2 V_3$ being ordered to appear on $V_1$ in the combination $\alpha \infty$, they would be ordered to appear in the form $\alpha \infty ^{\alpha - 1}$.

They would then stand thus: $V_0 V_1 V_2 V_3$.

Similarly, we might have $\frac{1}{\infty} \alpha^{\infty+1}$, the integral of $\alpha \infty$.

An interesting example for following out the processes of the engine would be such a form as

$$\int \frac{\alpha \infty}{\sqrt{\alpha \infty^2 - \beta \infty}}$$

or any other case of integration by successive reductions, where an integral which contains an operation repeated a times can be made to depend upon another which contains the same $a - 1$ or $a - 2$ times, and so on until by continued reduction we arrive at a certain ultimate form, whose value has then to be determined.

The methods in Arbigas's Cours des Dérivations are peculiarly fitted for the notation and the processes of the engine. Likewise the whole of the Combinatorial Analysis, which consists first in a purely numerical calculation of indices, and secondly in the distribution and combination of the quantities according to laws prescribed by them indices.

We will terminate these Notes by following up in detail the steps through which the engine could compute the Numbers of Bernoulli, this being (the form in which we shall deduce it) a rather complicated example of its powers. The simplest manner of composing these numbers would be from the direct expansion of

$$e^x - 1 = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \text{&c.}$$  \hspace{1cm} (1)
Multiplying every term by \((2n + 1)\), we have:

\[
0 = - \frac{1}{2} \left( \frac{2n}{2n + 1} \right) + B_2 \left( \frac{2n}{2n + 1} \right) + B_3 \left( \frac{2n - 2}{2n + 1} \right) + \cdots + B_{n-1} \left( \frac{2n - 2}{2n + 1} \right)
\]

which may be convenient to write under the general form:

\[0 = A_0 + A_1 + B_1 + A_2 + B_2 + \cdots + B_{n-1} - \frac{1}{2} \frac{2n}{2n + 1} \]

\[2A_n - n = \frac{d}{2} \quad \text{for any odd power of } n.
\]

It is with the independent formula \((8)\) that we have to do. Therefore it must be remembered that the conditions for the value of \(n\) are not modified, and that \(n\) is a perfectly arbitrary whole number. This circumstance, combined with the fact (which we may easily perceive) that whatever \(n\) is, every term of \((8)\) after the \((n + 1)\)th is zero, and that the \((n + 1)\)th term itself is always \(= B_{n+1} - \frac{1}{2} B_{n+1}\), enables us to find the value (either numerical or algebraical) of any number of Bernoulli \(B_{n+1}\) in terms of all the preceding ones, if we know the values of \(B_1, B_2, \ldots, B_{n-1}\). We appeal to this Note a Diagram and Table, containing the details of the computation for \(B_1, B_2, B_5\), being supposed given.

On attentively considering \((8)\), we shall likewise perceive that we may derive from it the numerical value of every number of Bernoulli in succession, from the very beginning, ad infinitum, by the following series of computations:

1st Series. Let \(n = 1\), and calculate \((8)\) for this value of \(n\). The result is \(B_1\).

2nd Series. Let \(n = 2\). Calculate \((8)\) for this value of \(n\), substituting the value of \(B_1\) just obtained. The result is \(B_2\).

3rd Series. Let \(n = 3\). Calculate \((8)\) for this value of \(n\), substituting the values of \(B_1, B_2\) before obtained. The result is \(B_3\). And so on to any extent.

The diagram represents the columns of the engine when just prepared.

* See the diagram at the end of these Notes.
zero upon it. (The sign at the top of $V_i$ would become $-$ during this process.)

Operation 7 will be unintelligible, unless it be remembered that if we were calculating for $n = 1$ instead of $n = 4$, Operation 6 would have completed the computation of $B_i$, itself; in which case the engine, instead of continuing its processes, would have to put $B_i$ on $V_1$ and then either to stop altogether, or to begin Operations 1, 2, 7 all over again for the value of $n = 1$; in order to begin or to continue the computation of $B_i$ (having however taken care, previous to this recommencement, to make the number on $V_1$ equal to zero, by the addition of unity to the former $n = 1$ on that column). Now Operation 7 must either bring out a result equal to zero (if $n = 1$), or a result greater than zero, as in the present case; and the engine follows the one or the other of the two courses just explained, contiguously on the one or the other result of Operation 7. In order fully to perceive the necessity of this experimental operation, it is important to keep in mind what was pointed out, that we are not treating a perfectly isolated and independent computation, but one out of a series of antecedent and prospective computations.

Cards 8, 9, 10 produce $1 - 2m_1 - 1 + B_i$. In Operation 9 we see an example of an upper index which again becomes a value after preceding values to zero. $V_i$ has successively been $V_i$, $V_i$, $V_i$, $V_i$, $V_i$, and, from the nature of the office which $V_i$ performs in the calculation, its index will continue to go through further changes of the same description, which, if examined, will be found to be regular and periodic.

Card 12 has to perform the same office as Card 7 did in the preceding section: since, if $m_1$ had been $2$, the 11th operation would have completed the computation of $B_i$.

Cards 13 to 20 make $A_i$. Since $A_i$ always consists of $2m_1 - 1$ factors, it will be seen that Cards 13, 14, 15, 16 make the second of these factors, and then multiply it with the first; and that 17, 18, 19, 20 make the third factor, and then multiply this with the product of the two former factors.

Card 23 has the office of Cards 11 and 7 to perform, since if $n = 2$, the 21st and 22nd operations would complete the computation of $B_i$. As our case is $B_i$, the computation will continue one more stage; and we must now direct attention to the facts that in order to compute $A_i$, it is merely necessary precisely to repeat the group of Operations 13 to 20; and then, in order to complete the computation of $B_i$, to repeat Operations 21, 22.

It will be perceived that every unit added to $n$ in $B_i$ entails an additional repetition of operations (13...20) for the computation of $B_i$. Not only are all the operations precisely the same however for every such repetition, but they require to be respectively supplied with numbers from the very same points of columns; with only the one exception of Operation 21; which will of course use $B_i$ (from $V_i$) instead of $B_i$ (from $V_i$). This identity in the columns which supply the respective numbers, must not be confounded with identity in the columns upon which they are given to the unit. Most of these values undergo alterations during a performance of the operations (13...20), and consequently the columns present a new set of values for the next performance of (13...23) to work on.

At the termination of the repetition of operations (13...23) in computing $B_i$, the alterations in the values of the Variables are such that $V_i = 2n - 4$ instead of $2 = 2$. $V_i = 2m_1 - 4$ instead of $1$.

$A_i + B_i + A_i + B_i + A_i + B_i$ is, in this state; the only remaining processes are first; to transfer the values of $V_i$ from $V_i$ to $V_i$; and secondly to reduce $V_i$, $V_i$, $V_i$ to zero, and to add $a_i$ to $V_i$ in order that the engine may be ready to commence computing $B_i$. Operations 24 and 25 accomplish these purposes. It may be thought anomalous that Operation 25 is represented as leaving the upper index of $V_i$ still as unity. But it must be remembered that these indices always begin anew for a separate calculation, and that Operation 25 places upon $V_i$, the first value for the new calculation.

It should be remarked, that when the group (18...23) is repeated, changes occur in some of the upper indices during the course of the repetition; for example, $a_i$ would become $V_i$ and $V_i$. We thus see that when $n = 1$, nine Operation-cards are used; that when $n = 2$, fourteen Operation-cards are used; and that when $n = 3$, twenty-five Operation-cards are used; but that no more are needed, however great $n$ may be; and not only this, but that these same twenty-five cards suffice for the successive computation of all the Numbers from $B_i$ to $B_i$, inclusive. With respect to the number of Variable-cards, it will be remembered, from the explanations in previous Notes, that an average of three such cards to each operation (not however to each Operation-card) is the estimate. According to this the computation of $B_i$ will require twenty-seven Variable-cards; $B_i$ cards make $A_i$ such cards; $B_i$ cards make $A_i$ such cards; and for every succeeding $B_i$ after $B_i$, there would be thirty-three additional Variable-cards (since each repetition of the group (18...23) adds eleven to the number of operations required for computing the previous $B$). But we must now consider, that when we come to the second stage, $B_i$, that is, when a cycle of operations (13...23) is, in that Operation 21 always requires one of its factors from a new column, and Operation 24 always puts its result in B(i).
ON Babbage's Analytical Engine.

A new column. But as these variations follow the same law at each repetition, (Operation 21) always requiring its factors from a column one in advance of that which it used the previous time, and Operation 22 always putting its result on the column one in advance that which received the previous result, they are easily provided for in arranging the recurring group (or cycle) of Variable-cards.

We may here remark that the average estimate of three Variable-cards coming into use to each establishment is not to be taken as an absolutely correct amount for all cases, and circumstances. Many special circumstances, either in the nature of a problem, or in the arrangements of the engine, under certain contingencies, influence and modify this average to a greater or less extent. But it is a very safe and correct general rule to go by. In the preceding case it will give us seventy-five Variable-cards as the total number which will be necessary for computing any B after B. This is very nearly the precise amount really used, but we cannot enter into the minute of the few particular circumstances which occur in this example (as indeed at some one stage or other of probably most computations) to modify slightly this number.

It will be obvious that the very same seventy-five Variable-cards may be repeated for the computation of every succeeding Number, not in the same principle as admits of the repetition of the thirty-three Variable-cards of Operations (13...23) in the computation of any one Number. Thus there will be a cycle of a cycle of Variable-cards.

If we now apply the notion of cycles, as explained in Note E, we may express the operations for computing the Numbers of Bernoulli in the following manner:

\[
\begin{align*}
(1-7), (24, 25) & \quad \text{gives } B_1 = 1 \text{st number; } (a \text{ being } 1) \\
(1-7), (8, 12), (24, 25) & \quad B_2 = 2\text{nd number } (a) \\
(1-7), (8, 12), (18, 28) & \quad B_3 = 3\text{rd number } (a) \\
(1-7), (8, 12), (24, 26) & \quad B_4 = 4\text{th number } (a) \\
(1-7), (8, 12), (24, 26) & \quad \text{et al. } (n) \\
\end{align*}
\]

Again,

\[
\begin{align*}
(1-7), (24, 25), \sum (a) = 1 & \quad \left\{ (1-7), (8, 12), \sum (n+2) (13, 28), (24, 35) \right\} \\
\end{align*}
\]

represents the total operations for computing every number in succession, from \( B_1 \) to \( B_{n-1} \) inclusive.

In this formula we see a merging cycle of the first order, and an ordinary cycle of the second order. The latter cycle in this case includes in it the varying cycle.

On inspecting the ten Working-Variables of the diagram, it will be perceived, that although the value on any one of them (excluding \( V_1 \) and \( V_2 \)) goes through a series of changes, the offices which each performs in the calculation fixed and invariable. Thus \( V_2 \) always preserves the assessor of the factors of any \( \Delta \); \( V_1 \), the denominator; \( V_1 \) always receives the \( (2n-2) \)th factor of \( A_{n-1} \); and \( V_4 \) the \( (2n-1) \)th of \( Y_0 \). Always always decides which of two courses the succeeding processes are to follow, by feeding the value of \( n \) through
Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

<table>
<thead>
<tr>
<th>Number of Operations</th>
<th>Variables used upon</th>
<th>Indication of change in the value on any Variable</th>
<th>Statement of Results</th>
<th>Working Variables</th>
<th>Result Variables</th>
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<td>$2n+1$</td>
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<td>$2n+1$</td>
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<td></td>
<td>$2n+1$</td>
</tr>
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<td>$2n+1$</td>
<td></td>
<td>$2n+1$</td>
</tr>
<tr>
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<td>$2n+1$</td>
<td></td>
<td>$2n+1$</td>
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<td>$2n+1$</td>
<td></td>
<td>$2n+1$</td>
</tr>
<tr>
<td>24</td>
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<td>$2n+1$</td>
<td></td>
<td>$2n+1$</td>
</tr>
<tr>
<td>25</td>
<td>$V_2 + V_4$</td>
<td>$V_2 + V_4$</td>
<td>$2n+1$</td>
<td></td>
<td>$2n+1$</td>
</tr>
</tbody>
</table>

Here follows a repetition of Operations thirteen to twenty-three.
My dear Lady Lovelace,

If you are as fastidious about acts of your friends as you are about those of your pen, I much fear I shall equally lose your friendship and your notes. I like much the improved form of the Bernouille* Note but can judge of it better when I have the diagram and Notation. [*Bernouelli?]

I am very reluctant to return the admirable and philosophic view of the Abreal Engine contained in Note A. Pray do not alter it and do let me have it returned on Monday. I send also the rest of Note D. There is still one trifling misapprehension about the Variable cards — A Variable card may order any number of Variables to receive the same number upon theirs at the same instant of time — But a Variable card never can be directed to order more than one Variable to be given off at once because the mill could not receive it the mechanism would not permit it. All this it was impossible for you to know by intuition and the more I read your notes the more surprised I am at them and regret not having earlier explored so rich a vein of the noblest metal.

The account of them stands thus:

A Sent to Lady L.                     F Retained by Lady L.  
B With C.B.                           G Where is it gone?  
C Ditto                              H With C.B.  
D Sent to Lady L.                     
E With C.B.

I have not seen Mr. Wheatstone and am ashamed to write until I can positively put the whole of the notes into his hands. 

I will attend your commands tomorrow —

And am,

Ever most truly yours,

C. BABBAGE

1 Dorset St.  
Manchr Sq.  
2nd July 1843
Ockham, Sunday 6 o'clock, 2 July

I have worked incessantly, & most successfully, all day. You will admire the Table & Diagram extremely.

They have been made out with extreme care, & all the indices most minutely & scrupulously attended to. Lord L___ is at this moment kindly asking it all over for me.

I had to do it in pencil.

You must bring all the Notes with you tomorrow; as I have observations to make on each one; & especially on this final one H.

There never was a Note G. I do not know why I chose H instead of G, & thus insulted the latter worthy letter.

I cannot imagine what you mean about the Variable-Cards; since I never either supposed in my own mind, that one Variable-card could give off more than one Variable at a time; nor have (as far as I can make out) expressed such an idea in any passage whatever.

I cannot find what I fancied I had put in Note A; so I return it whole & sound, for your speedy relief.

I send back note D. You will find the only alteration I wished to make pinned over; in the upper part of Sheet 2.

So I now retain nothing but Note F, which I shall give you tomorrow.

Lord L___ has put up, I find, in a separate cover, all that belongs to Note H. (He is quite enchanted with the beauty & symmetry of the Table & Diagram). No – I find I can put in Note D with H.
I had to do it in pencil.

For want, being all the notes with you tomorrow so I have observations to make on each one, especially on this final. I have reason was a note.

As I do not know why I chose H instead of G, I then inserted the latter worthy letter. I cannot imagine.

What you mean about the #Askable Card? Since I mean either subject in my own mind, that is an Askable card could give off more than one answer with a term I have (as far as I can make out) correspond to such an idea in any passage whatever.

I cannot find what I fancied I had left in A & so I return it while it sounds for you.
I read back Note D only.

You will find the alteration I wished to make in the upper part of Sheet 2.

I am ready to receive any further note. 

I shall give you tomorrow and I have put up, I find, in a separate cover, all that belongs to Note H. (She is quite enchanted with the beauty and symmetry of the Table of Diagrams). 

No— I find I can put in Note D with H.
Letter from Augusta, Ada, Countess of Lovelace to Mr. C. Babbage

Thursday, 4 July 1843, Ockham

My Dear Babbage. I now write to you expressly on three points; which I have very fully & leisurely considered during the last 18 hours; & think of sufficient importance to induce me to send a servant up so that you may have this letter by half after six this evening. The servant will leave Town tomorrow morning early, but will call for anything you may have for me, at eight o’clock in the morning before he goes.

Firstly: The few lines I enclosed you last night about the connection of (8) with the famous Integral, I by no means intend you to insert, unless you fully approve the doing so.

It is perhaps very dubious whether there is any sufficient pertinence in noticing at all that (8) is an Integral. Is not every formula an Integral of something? ( . . . we may not always be able to determine the form after something as in this case); and is this consequently, any way palpable an important purpose announced in noticing that (8) is an Integral? Such notice would rather seem to imply that this formulae mean not Integral, & that (8) is peculiar, either in being an Integral at all, or else in being the Integral of a very important & peculiar formula. In short the pros & cons from the insertion, seem to me to depend mainly upon the two following questions:

Is (8) more pre-eminently an Integral rather than formulae?

If not, then is the form of which it is an Integral, in any way a very remarkable form?

Should the first question be answered in the negative, still the second question may justify the assertion, if answered affirmatively. But if both are negative, I think the insertion irrelevant.

Secondly: Lord L___ suggests my signing the translation & the Notes; by which he means, simply putting at the end of the former: ‘translated by A.A.L.’; & adding to each note the initials A.A.L.

It is not my wish to proclaim who has written it; at the same time that I rather wish to append anything that may tend hereafter to individualise, & identify it, with other productions of the said A.A.L.

My third topic, tho my last, is our most anxious & important: I have yesterday evening & this morning very amply analysed the question of the number of Variable-Cards, as mentioned in the final Note H (or G?). And I find that you & I between us made a mess of it (for which I can perfectly account, in a very natural manner). I enclose what I wish to insert instead of that which is now there. I think the present wrong passage is only about eight or ten lines, & is I believe on the second of the three great sheets which are to follow the Diagram.

The fact is that if my own exposition about the Variable-Cards on Note D, had been strictly followed by myself, in Note H, this error would not have occurred. The confusion has arisen simply from the circumstance of applying to the Variable-Cards, facts which relate to the Operation-Cards.

In Note D, it is very well & lucidly demonstrated that every single Operation, demands the use of at least three Variable-Cards. It does not signify whether the operations be in cycles or not. A million successive additions +, +, +, &c &c, &c, would each demand the use of three new Variable-Cards, under ordinary circumstances. In Note H, the erroneous lines are found on the hasty supposition that the cycle, or recurring group, of Operation-Cards (13 . . . 23) will be fed by a cycle, or recurring group, of Variable-Cards.

I enclose what I believe it ought to be:

If already gone to the printer, we must alter that passage in the proofs, unless you could call at the printers & there paste over the amendment.

I can scarcely describe to you how very ill & harassed I felt yesterday. Pray excuse any abruptness or other unpleasantness of manner, if there were any.

I am breathing well again today, & am much better in all respects; owing to Dr L’s remedies. He certainly does seem to understand the case, I mean the treatment of it, which is the main thing.

As for the theory of it, he says truly that time & Providence alone can develop that. It is so anomalous an affair altogether. A Singular Function, in very deed!

Think of my having to walk, (or rather run), to the Station, in half an hour last evening; while I suppose you were feasting & flirting in luxury & ease at your dinner.

It must be a very pleasant merry sort of thing to have a Fairy in one’s service, mind & limbs! – I envy you! – I, poor little Fairy, can only get dull heavy mortals, to wait on me! – Ever Yours A.L.

Transcript from handwritten original, British Library / Transkription des handschriftlichen Originals, British Library
Ada Lovelace,  
The Rainbow, 1851

Sonnet  
The Rainbow.

Bow down in hope, in thanks, all ye who mourn; –  
Where’in that peerless arch of radiant hues  
Surpassing earthly tints, – the storm subdues!  
Of nature’s strife and tears ’tis heaven-born,  
To soothe the sad, the sinning and the forlorn; –  
A lovely loving token; to infuse;  
The hope, the faith, that pow’r divine endures  
With latent good, the woes by which we’re torn.

"Tis like a sweet repentance of the skies;  
To beckon all those by sense of sin opprest,  
And prove what loveliness may spring from sighs!  
A pledge: – that’s deep implanted in the breast  
A hidden light may burn that never dies,  
But bursts thro’ clouds in purest hues exprest!

"Tis like a sweet repentance of the skies,  
To beckon all by sense of sin opprest, –  
Proclaiming loveliness from tears and sighs!  
A pledge: – that’s deep implanted in the breast  
A hidden light may burn that never dies,  
But bursts thro’ clouds in purest hues exprest!

Transcript from handwritten original, British Library / Transkription des handschriftlichen Originals, British Library
100 Notes – 100 Thoughts / 100 Notizen – 100 Gedanken

Nº055: Ada Lovelace

dOCUMENTA (13), 9/6/2012 – 16/9/2012

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Member of Core Agent Group, Head of Department / Mitglied der Agenten-Kerngruppe, Leiterin der Abteilung: Chus Martínez
Head of Publications / Leiterin der Publikationsabteilung: Bettina Funcke
Managing Editor / Redaktion und Lektorat: Katrin Sauerländer
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Graphic Design and Typesetting / Grafische Gestaltung und Satz: Leftloft
Typography / Schrift: Glypha, Plantin
Production / Verlagsherstellung: Christine Emter
Reproductions / Reproduktionen: weyhing digital, Ostfildern
Paper / Papier: Pop’Set, 240 g/m², Munken Print Cream 15, 90 g/m²
Manufacturing / Gesamtherstellung: Dr. Cantz’sche Druckerei, Ostfildern

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documenta und Museum Fridericianum
Veranstaltungs-GmbH
Friedrichplatz 18, 34117 Kassel
Germany / Deutschland
Tel. +49 561 70727-0
Fax +49 561 70727-39
www.documenta.de
Chief Executive Officer / Geschäftsführer: Bernd Leifeld

Published by / Erschienen im
Hatje Cantz Verlag
Zeppelinstraße 32, 73760 Ostfildern
Germany / Deutschland
Tel. +49 711 4405-200
Fax +49 711 4405-230
www.hatjecantz.com

ISBN 978-3-7757-2804-8 (Print)
ISBN 978-3-7757-3084-6 (E-Book)

Printed in Germany

Funded by the German Federal Cultural Foundation
Ada Lovelace

Introduction / Einführung:
Joasia Krysa